Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4MA1/1HR**

Mathematics A
PAPER: 1HR
Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

1/1/1/
P 6 8 7 8 9 A 0 1 2 8

Pearson

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

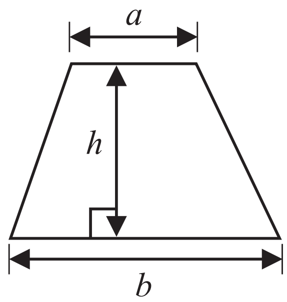
Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

The quadratic equation

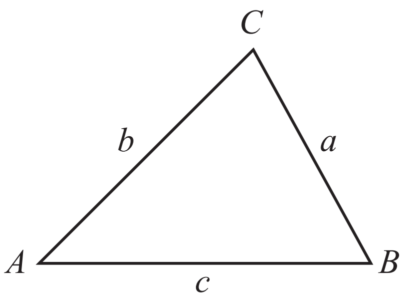
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \frac{1}{2}(a + b)h$



Trigonometry



In any triangle *ABC*

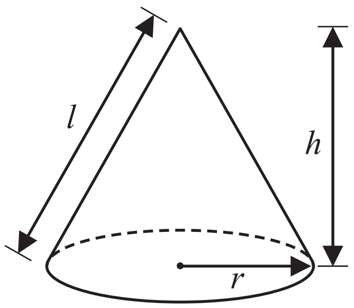
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

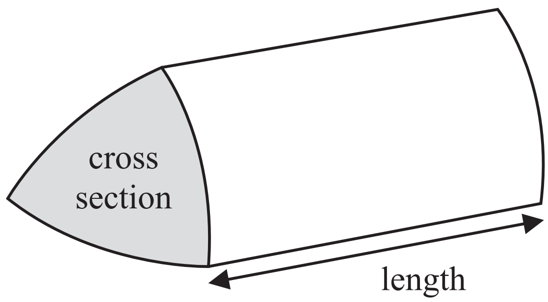
Area of triangle $= \frac{1}{2}ab\sin C$

Volume of cone $= \frac{1}{3}\pi r^2 h$

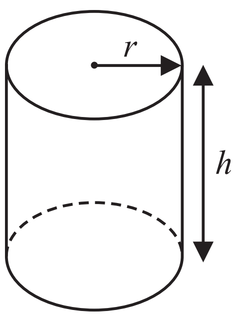
Curved surface area of cone $= \pi rl$



Volume of prism
= area of cross section × length

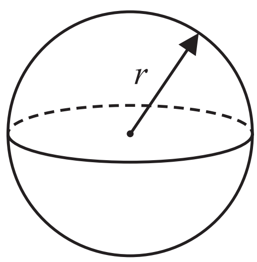


Volume of cylinder $= \pi r^2 h$
Curved surface area
of cylinder $= 2\pi rh$



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

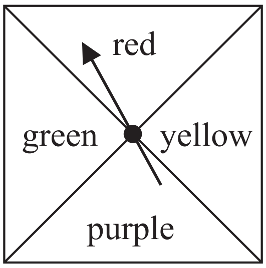


Answer ALL TWENTY FIVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Here is a biased spinner.



When the spinner is spun once, the probabilities that it lands on red or on yellow or on green are given in the table.

Colour	red	yellow	purple	green
Probability	0.25	0.2	0·35	0.2

(a) Work out the probability that the spinner lands on red or on yellow.

$P(R) + P(Y) = 0\cdot25 + 0\cdot2$

$= 0\cdot45$ 1

0·45

(1)

Yang is going to spin the spinner 300 times.

(b) Work out an estimate for the number of times the spinner will land on purple.

$1 - (0\cdot25 + 0\cdot2 + 0\cdot2)$ 1

$= 0\cdot35$

$0\cdot35 \times 300 = 105$ 1 1

105

(3)

(Total for Question 1 is 4 marks)

2 In a warehouse there are two types of shelves, type **R** and type **S**.

These two types of shelves are arranged into shelving units that form a sequence of patterns.

Here are the first three terms in the sequence.

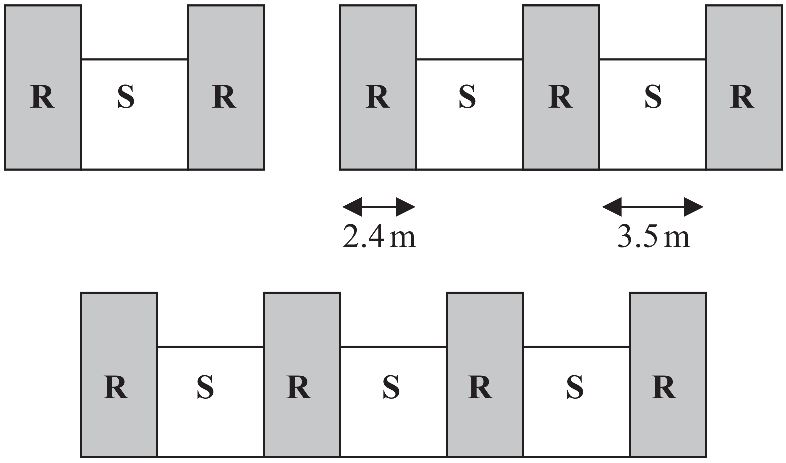


Diagram **NOT** accurately drawn

$2R + 1S$

The width of each type **R** shelf is 2.4 m and the width of each type **S** shelf is 3.5 m

(a) Work out the total width of a shelving unit that has 6 type **R** shelves.

[6 R shelves + 5 S shelves]

$= 6 \times R + 5 \times S$

$= (6 \times 2.4) + (5 \times 3.5)$ 1

$= 14.4 + 17.5$

$= 31.9$ 1

31.9 m

(2)

A shelving unit has n type **R** shelves.
The total width of this shelving unit is W metres.

(b) Find an expression for W in terms of n
Give your answer in its simplest form.

$T_1 = 2R + S$

$T_2 = 3R + 2S$

$T_n = nR + (n-1)S$

$W = n(2.4) + (n-1)(3.5)$

$= 2.4n + 3.5n - 3.5$ 1

$W = 5.9n - 3.5$ 1

$W =$ $5.9n - 3.5$

(2)

(Total for Question 2 is 4 marks)

3 Here are five cards.

Each card has a number written on it.

15	7	–2	23	x

The mean of the five numbers is 12

Work out the value of x

$$\text{Mean} = \frac{15 + 7 + (-2) + (23) + x}{5} = 12 \quad 1$$

$$43 + x = 12(5) \quad 1$$

$$x = 60 - 43$$

$$= 17 \quad 1$$

$x =$ 17

(Total for Question 3 is 3 marks)

4 The language department of a college has 180 students.
Each student studies exactly one of French, German, Italian or Spanish.

15 students study French.
45% of the students study German.

Express the percentage of students studying Italian or Spanish as a percentage of those studying French or German.

% studying French $= \frac{15}{180} \times 100\% = 8.33\%$ (1)

German $= 45\%$

% of student studying French and German:

$45\% + 8.33\% = 53.33\%$

% of student studying Italian or Spanish:

$100\% - 53.33\% = 46.67\%$ (1)

$\frac{46.67\%}{53.33\%} \times 100\% = 87.5\%$ (1) (1)

87.5 %

(Total for Question 4 is 4 marks)

5 (a) Expand $3c^3(c + 4)$

$3c^4 + 12c^3$

$3c^4 + 12c^3$ 2

(2)

(b) (i) Factorise $x^2 + 8x - 9$

$x^2 + 8x - 9$

$(x - 1)(x + 9)$

2

$(x-1)(x+9)$

(2)

(ii) Hence, solve $x^2 + 8x - 9 = 0$

$(x-1)(x+9)$

$x = 1$ or $x = -9$

$1, -9$ 1

(1)

(Total for Question 5 is 5 marks)

6 Show that $2\frac{2}{3} + 3\frac{3}{4} = 6\frac{5}{12}$

$$a\frac{b}{c} = \frac{c \times a + b}{c}$$

$$\text{LHS} : \frac{8 \times 4}{3 \times 4} + \frac{15 \times 3}{4 \times 3} \quad (1)$$

$$= \frac{32}{12} + \frac{45}{12} \quad (1)$$

$$= \frac{77}{12} \quad (1)$$

$$= 6\frac{5}{12} \quad \text{(shown)}$$

$$\begin{array}{r} 6 \\ 12 \overline{)\,77} \\ -\,72 \\ \hline 5 \end{array}$$

(Total for Question 6 is 3 marks)

7 The diagram shows a solid cylinder made from iron.

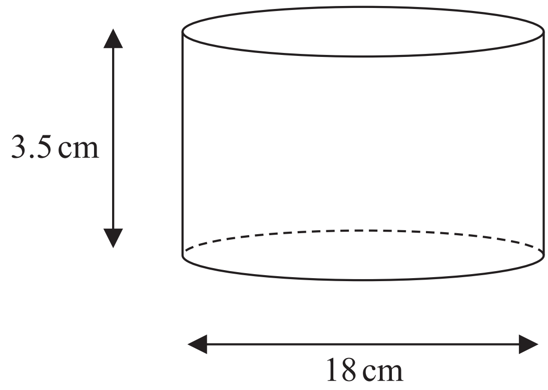


Diagram **NOT** accurately drawn

The cylinder has diameter 18 cm and height 3.5 cm
The mass of the cylinder is 7.04 kg

Work out the density of the iron.
Give your answer in g/cm³ correct to 2 significant figures.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$\pi \times r^2 \times h$

kg $\xrightarrow{\times 1000}$ g
g $\xrightarrow{\div 1000}$ kg

volume of cylinder $= \pi \times \left(\frac{18}{2}\right)^2 \times 3.5$

$= 890.64\ldots$ 1

density $= \dfrac{7.04 \times 1000}{890.64\ldots}$ – convert to g 1

$= 7.9 \text{ g/cm}^3$ 1

7.9 .... g/cm³

(Total for Question 7 is 3 marks)

8 Jane bought a new car for \$18 000
The car depreciates in value by 15% each year.

Work out the value of the car at the end of 4 years.
Give your answer correct to the nearest \$

Value each year : (100% − 15%) of value

= 85%

Value at the end of 4 years : $18\,000 \times \left(\frac{85}{100}\right)^4$ (2)

= 9396 (1)

\$ 9396

(Total for Question 8 is 3 marks)

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9 Solve the inequality $3 - 4x \leqslant 11$

$3 - 4x \leqslant 11$

$3 - 11 \leqslant 4x$

$-8 \leqslant 4x$ (1)

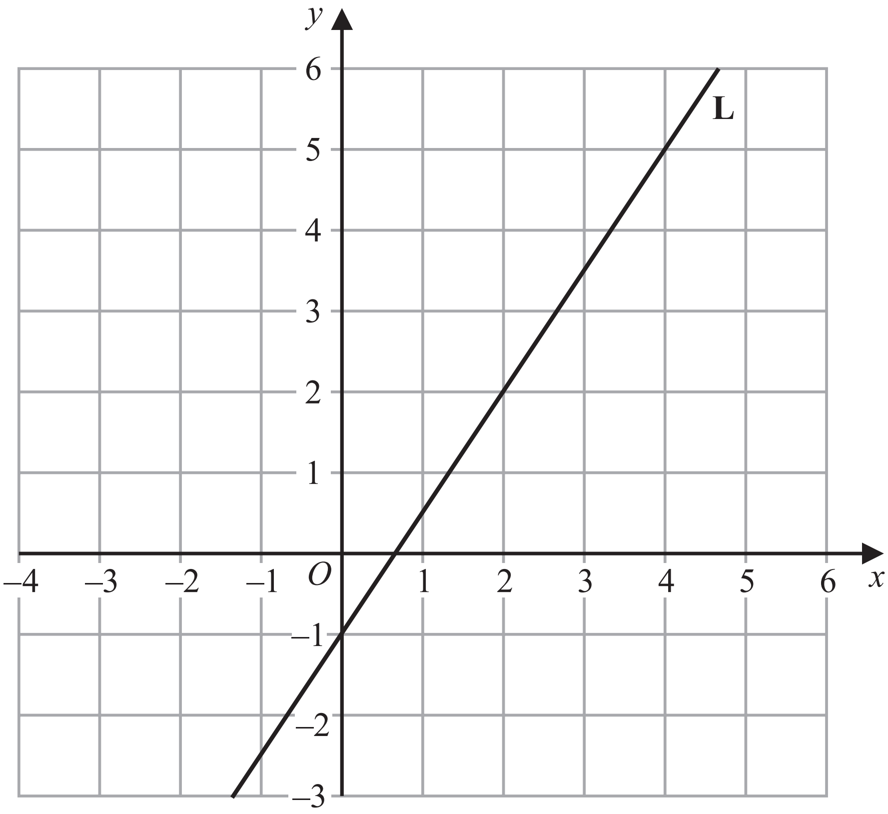
$\frac{-8}{4} \leqslant x$

$-2 \leqslant x$ (1)

$x \geqslant -2$

(Total for Question 9 is 2 marks)

10 Line **L** is drawn on the grid.



Find an equation for **L**

Give your answer in the form $y = mx + c$

c = y-intercept = -1 1

gradient = $\frac{5-(-1)}{4-0}$

$= \frac{6}{4} = \frac{3}{2}$ 1

$y = \frac{3}{2}x - 1$ 1 — $y = mx + c$

$y = \frac{3}{2}x - 1$

(Total for Question 10 is 3 marks)

11 The diagram shows a quadrilateral *ABCD*

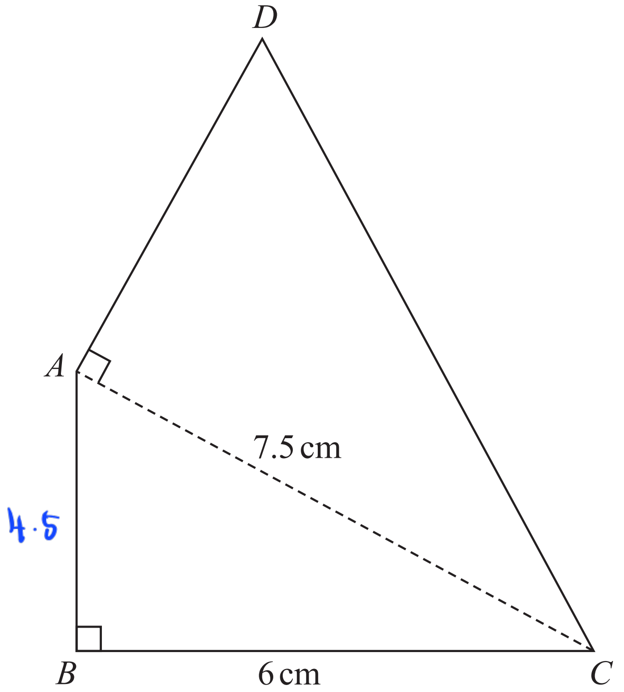


Diagram **NOT** accurately drawn

In the diagram, *ABC* and *DAC* are right-angled triangles.

$BC = 6$ cm $\qquad AC = 7.5$ cm

The area of quadrilateral *ABCD* is 31.5 cm^2

Work out the length of *AD*

By using Pythagoras' theorem :

length AB $= \sqrt{7.5^2 - 6^2}$ (1)

$= 4.5$ cm (1)

Area of triangle ABC : $\frac{1}{2} \times 6 \times 4.5 = 13.5$ cm^2 (1)

Area of triangle ADC : $31.5 - 13.5 = 18$ cm^2 (1)

$\frac{1}{2} \times AD \times 7.5 = 18$

$AD = \frac{18}{7.5} \times 2$ (1)

$= 4.8$ cm (1)

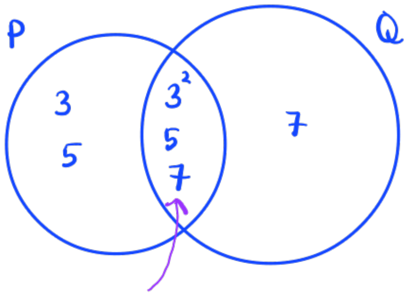
Question 11 continued.

4.8 cm

(Total for Question 11 is 6 marks)

12 $P = 3^3 \times 5^2 \times 7$
$Q = 3^2 \times 5 \times 7^2$

(a) Write down the highest common factor (HCF) of P and Q



HCF of P and Q = $3^2 \times 5 \times 7$

$3^2 \times 5 \times 7$ (1)

(1)

$P = 3^3 \times 5^2 \times 7$
$Q = 3^2 \times 5 \times 7^2$

(b) Work out the value of $P^3 \times Q$
Give your answer in the form $3^x \times 5^y \times 7^z$ where x, y and z are positive integers.

$P^3 = (3^3 \times 5^2 \times 7)^3$
$= 3^9 \times 5^6 \times 7^3$

$P^3 \times Q = (3^9 \times 5^6 \times 7^3) \times (3^2 \times 5 \times 7^2)$
$= 3^9 \times 3^2 \times 5^6 \times 5 \times 7^3 \times 7^2$
$= 3^{9+2} \times 5^{6+1} \times 7^{3+2}$
$= 3^{11} \times 5^7 \times 7^5$ (2)

$3^{11} \times 5^7 \times 7^5$

(2)

(Total for Question 12 is 3 marks)

13 Here is the number of runs scored by a baseball team in each of its 15 games this season.

The number of runs have been arranged in order of size.

lower quartile | | upper quartile

0 1 1 3 5 6 7 7 8 9 9 12 12 15 16

Q_1 | median | Q_3

Work out the interquartile range of the number of runs.

Q_1 = median of lower quartile

Q_3 = median of upper quartile

Interquartile range $= Q_3 - Q_1$

$= 12 - 3$ (1)

$= 9$ (1)

9

(Total for Question 13 is 2 marks)

14 Solve the simultaneous equations

$$3x - 5y = 25 \quad - ①$$
$$4x + 3y = 14$$

Show clear algebraic working.

$$x = \frac{14-3y}{4} \quad - ②$$

Substitute ② into ① :

$$3\left(\frac{14-3y}{4}\right) - 5y = 25 \quad ①$$

$$3(14-3y) - 5y(4) = 25(4)$$

$$42 - 9y - 20y = 100$$

$$-29y = 100 - 42$$

$$-29y = 58$$

$$y = \frac{58}{-29} = -2 \quad ①$$

Substitute $y = -2$ into ②

$$x = \frac{14-3(-2)}{4} \quad ①$$

$$= 5$$

$x =$ 5 ①

$y =$ -2

(Total for Question 14 is 4 marks)

15

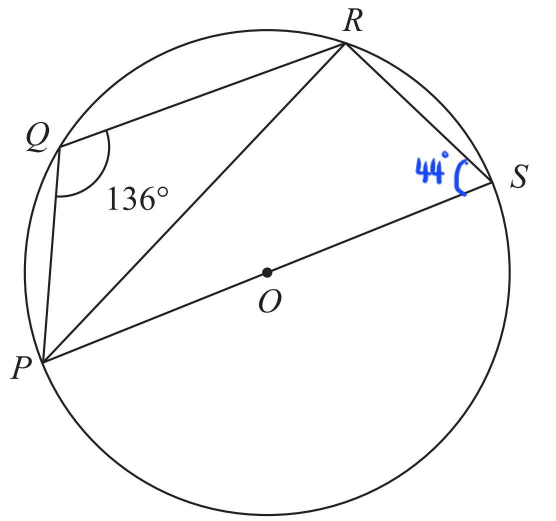


Diagram **NOT** accurately drawn

P, Q, R and S are points on a circle with centre O

PS is a diameter of the circle.

Angle $PQR = 136°$

Work out the size of angle RPS

angle PSR = 180° - 136°

= 44° (opposite angles in a cyclic quadrilateral sums up to 180°) 1

angle PRS = 90° (angle inside a semicircle is always 90°)

angle RPS = 180° - 44° - 90° 1

= 46° (angles in a triangle add up to 180°) 1

46 °

(Total for Question 15 is 3 marks)

16 (a) Expand and simplify $(3x-1)(x+2)(3x+1)$

Expand first 2 terms :

$(3x-1)(x+2) = 3x^2+6x-x-2$

$= 3x^2+5x-2$ (1)

Expand remaining term :

$(3x^2+5x-2)(3x+1) = 9x^3+3x^2+15x^2+5x-6x-2$ (1)

$= 9x^3+18x^2-x-2$ (1)

$9x^3+18x^2-x-2$

(3)

(b) Simplify fully $\left(\dfrac{2x^5}{8xy^2}\right)^{-2}$

Solve inside bracket first :

$\left[\dfrac{2x^5}{8xy^2}\right] = \left[\dfrac{x^4}{4y^2}\right]$

$\left[\dfrac{x^4}{4y^2}\right]^{-2}$ (1) $= (x^4)^{-2} \times (4^{-1})^{-2} \times (y^{-2})^{-2}$ (1)

$= x^{-8} \times 4^2 \times y^4$

$= \dfrac{16y^4}{x^8}$ (1)

$\dfrac{16y^4}{x^8}$

(3)

(Total for Question 16 is 6 marks)

17 Here is a parallelogram $PQRS$, in which angle SPQ is acute.

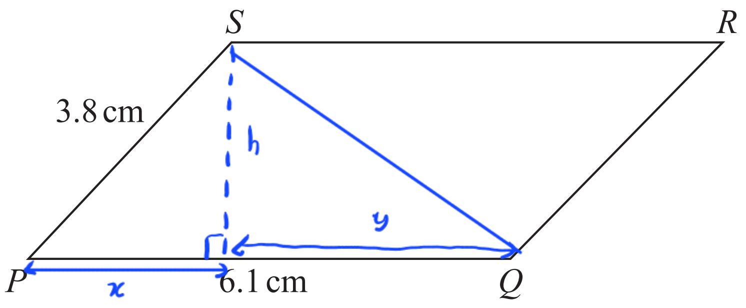


Diagram **NOT** accurately drawn

$PQ = 6.1$ cm $\quad\quad PS = 3.8$ cm

Area of parallelogram = base × height

The area of the parallelogram is 18 cm^2

Work out the length of QS
Give your answer correct to 3 significant figures.

Area of parallelogram $= 18 = 6.1 \times h$

$h = \frac{18}{6.1} = 2.95....$ (1)

Finding length x by Pythagoras' theorem :

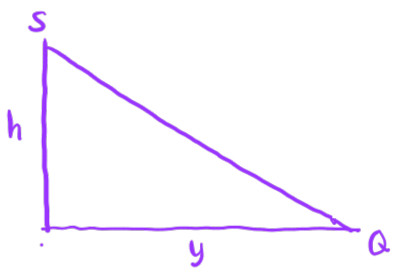
$x = \sqrt{3.8^2 - 2.95^2...}$

$= 2.394....$ (1)

length $y = 6.1 - 2.394$

$= 3.7057....$ (1)

Finding length QS :



$QS = \sqrt{h^2 + y^2}$

$= \sqrt{(2.95)^2 + (3.7057..)^2}$ (1)

$= 4.74$ (1)

4.74 .... cm

(Total for Question 17 is 5 marks)

18 The diagram shows a cube *ABCDEFGH* with sides of length 6 cm.

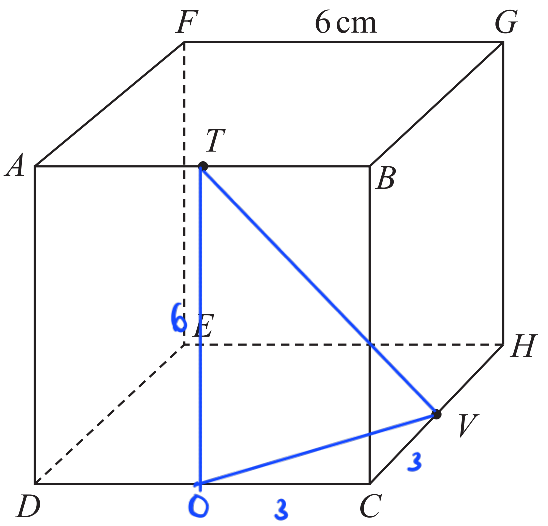
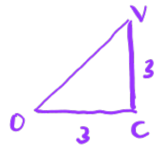


Diagram **NOT** accurately drawn

T is the midpoint of *AB* and *V* is the midpoint of *CH*

Work out the distance from *T* to *V* in a straight line through the cube.
Give your answer in the form $\sqrt{a}$ cm where *a* is an integer.

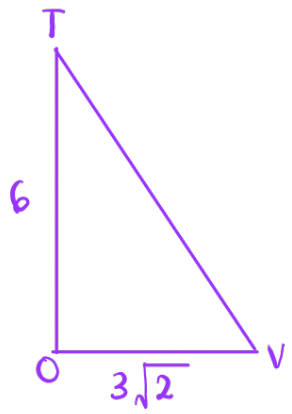
Finding length OV :



$OV = \sqrt{3^2 + 3^2}$ (1)

$= 3\sqrt{2}$ cm (1)

Finding length TV :



$TV = \sqrt{6^2 + (3\sqrt{2})^2}$ (1)

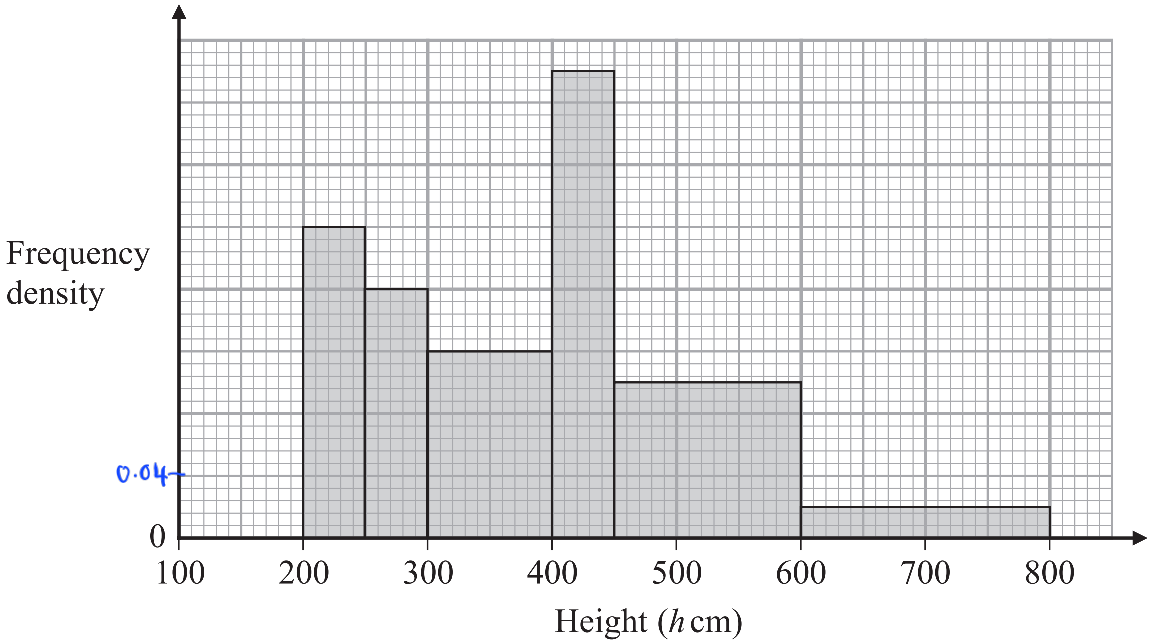
$= \sqrt{54}$ (1)

where $a = 54$

$\sqrt{54}$ .... cm

(Total for Question 18 is 4 marks)

19 The histogram gives information about the height, h cm, of each tree in part of a forest.



There are no trees for which $h \leqslant 200$ and for which $h > 800$

The number of trees for which $300 < h \leqslant 400$ is 8 fewer than the number of trees for which $400 < h \leqslant 500$

Work out an estimate for the number of trees in this part of the forest that have a height greater than 500 cm.

frequency = frequency density x width of class

Finding value of frequency density :

b = square box of 5x5

frequency of tree with h = 300 to 400 : 3b x 100 = 300b

frequency of tree with h = 400 to 450 : 7.5b x 50 = 375 b

h = 450 to 500 : 2.5b x 50 = 125 b

300 b = (375 b + 125 b) - 8

300 b = 500 b - 8

500 b - 300 b = 8

200 b = 8 (1)

b = 0.04 – 1 square box of 5x5 = 0.04

From h = 500 to 600 : 0.1 x 100 = 10

600 to 800 : 0.02 x 200 = 4 (1)

10 + 4 = 14 (1)

14

(Total for Question 19 is 3 marks)

20 The diagram shows two similar metal statues.

Diagram **NOT** accurately drawn

A **B**

The volume of statue **B** is 20% less than the volume of statue **A**

The surface area of statue **B** is k% less than the surface area of statue **A**

Work out the value of k
Give your answer correct to 3 significant figures.

Volume of B = 0.8 Volume of A

$$\frac{\text{Volume of B}}{\text{Volume of A}} = 0.8$$

To find length scale factor:

$$\left(\frac{\text{Volume of B}}{\text{Volume of A}}\right)^{1/3} = \frac{\text{Length of B}}{\text{Length of A}} = (0.8)^{1/3} \quad 1$$

To find area scale factor:

$$\frac{\text{Area of B}}{\text{Area of A}} = \left(0.8^{1/3}\right)^2$$

$$= 0.8^{2/3} = 0.8617\ldots \quad 1$$

Surface Area of B = $0.8617\ldots \times 100\%$ of surface Area of A

= 86.2% of surface Area of A

= less than $(100 - 86.2)\%$ of A 1

= less than 13.8% of A 1

$k =$ 13.8

(Total for Question 20 is 4 marks)

21 Express $\dfrac{3+\sqrt{8}}{\left(\sqrt{2}-1\right)^2}$ in the form $p+\sqrt{q}$ where p and q are integers.

Show each stage of your working clearly.

$$\frac{3+\sqrt{8}}{(\sqrt{2}-1)(\sqrt{2}-1)}$$

$$= \frac{3+\sqrt{8}}{2-2\sqrt{2}+1} \quad (1)$$

$$= \frac{3+\sqrt{8}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} \quad (1)$$

$$= \frac{9+6\sqrt{2}+3\sqrt{8}+2\sqrt{16}}{9-4(2)}$$

$$= \frac{9+6\sqrt{2}+3(2\sqrt{2})+2(4)}{1} \quad (1)$$

$$= 17+12\sqrt{2}$$

$$= 17+\sqrt{12^2 \times 2} = 17+\sqrt{288} \quad (1)$$

$17+\sqrt{288}$

(Total for Question 21 is 4 marks)

Turn over for Question 22

22 The diagram shows a sketch of part of the curve with equation $y = x^2 - \frac{p}{x}$ where p is a positive constant.

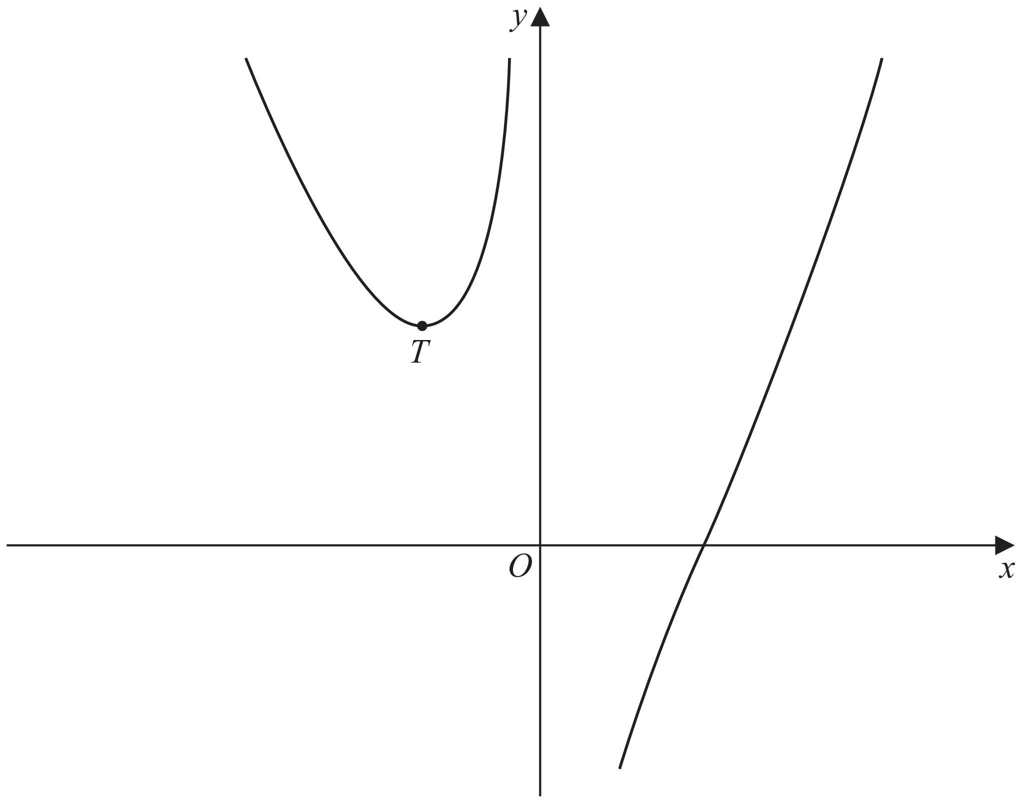


Diagram **NOT** accurately drawn

For all values of p, the curve has exactly one turning point and this turning point is a minimum shown as the point T in the sketch.

For the curve where the x coordinate of T is -3

(a) find the value of p

turning point: $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 2x + \frac{p}{x^2} = 0$ (1) (1)

When $x = -3$

$2(-3) + \frac{p}{(-3)^2} = 0$ (1)

$p = 54$ (1)

$p =$ 54

(4)

The line with equation $y = k$ is a tangent to the curve with equation $y = x^2 - \dfrac{16}{x}$

(b) Find the value of k

tangent $= \dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = 2x + \dfrac{16}{x^2}$

$2x + \dfrac{16}{x^2} = 0$ 1

$2x^3 + 16 = 0$

$x^3 = \dfrac{-16}{2}$

$x^3 = -8$

$x = \sqrt[3]{-8}$

$= -2$ 1

$y = (-2)^2 - \dfrac{16}{-2}$

$= 4 + 8$

$= 12$ 1

$k = y = 12$

$k =$ 12

(3)

(Total for Question 22 is 7 marks)

Turn over for Question 23

23 (a) Express $2x^2 - 12x + 3$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

$2(x^2 - 6x) + 3$ ①

$2[(x-3)^2 - 9] + 3$ ①

$2(x-3)^2 - 18 + 3$

$2(x-3)^2 - 15$ ①

where $a = 2$, $b = -3$ and $c = -15$

$2(x-3)^2 - 15$

(3)

The curve **C** has equation $y = 2(x + 4)^2 - 12(x + 4) + 3$

The point M is the minimum point on **C**

(b) Find the coordinates of M

$y = 2x^2 + 16x + 32 - 12x + 48 + 3$

$= 2x^2 + 4x + 83$

$\frac{dy}{dx} = 4x + 4 = 0$

$x = \frac{-4}{4} = -1$ ①

$y = 2(-1+4)^2 - 12(-1+4) + 3$

$= 2(3)^2 - 12(3) + 3$

$= 18 - 36 + 3$

$= -15$ ①

(-1 , -15)

(2)

(Total for Question 23 is 5 marks)

24 Elliot has x counters.

Each counter has one red face and one green face.

Elliot spreads all the counters out on a table and sees that the number of counters showing a red face is 5

Elliot then picks at random one of the counters and turns the counter over.
He then picks at random a second counter and turns the counter over.

The probability that there are still 5 counters showing a red face is $\frac{19}{32}$

Work out the value of x
Show clear algebraic working.

To get 5 counters still showing red face :

① First pick (R) + second pick (G)

② First pick (G) + second pick (R)

① $\frac{5}{x} \times \frac{(x-4)}{x} = \frac{5x-20}{x^2}$ ①

② $\frac{(x-5)}{x} \times \frac{6}{x} = \frac{6x-30}{x^2}$

① + ② $= \frac{19}{32}$

$\frac{5x-20+6x-30}{x^2} = \frac{19}{32}$ ①

$11x-50 = \frac{19}{32}(x^2)$

$32(11x-50) = 19x^2$

$19x^2 - 352x + 1600 = 0$ ①

$(19x-200)(x-8) = 0$ ①

$x = \frac{200}{19}$ or $x = 8$

$x = $ 8 ①

$\therefore x = 8$ since $\frac{200}{19}$ is not a whole number

(Total for Question 24 is 5 marks)

25 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.

The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2a + (10-1)d]$

$= 10a + 45d$

$S_5 = \frac{5}{2}[2a + (5-1)d]$

$= 5a + 10d$

$S_{10} = 4 \times S_5$

$10a + 45d = 4(5a + 10d)$ (1)

$10a + 45d = 20a + 40d$

$10a = 5d$

$d = 2a$ — (1) (1)

$T_n = a + (n-1)d$

$T_8 = 45 = a + (8-1)d$

$45 = a + 7d$ — (2) (1)

substitute (1) into (2) :

$45 = a + 7(2a)$ (1)

$45 = 15a$

$a = 3$ (1)

∴ First term, $a = 3$.

3

(Total for Question 25 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS